

Cerenkov radiation within the ionospheric anisotropic plasma

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Abstract : Cerenkov radiation for a point charge moving with uniform velocity within the ionospheric plasma has been investigated in presence of geomagnetic field, time-varying irregularities and motion of heavy ions. The expression for frequency spectrum of radiated energy is derived. From numerical analysis, the direction of Cerenkov radiation has been shown graphically along with the results of an earlier work.

Keywords : Cerenkov radiation, ionospheric plasma, frequency spectrum.

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1. Introduction

Various aspects of the problems of radiation by a charged particle moving with a uniform velocity within a plasma in presence of external magnetic field have been examined [1–7]. The radiation characteristics of a point charge moving with uniform velocity along the direction of the static magnetic field in an anisotropic plasma are studied. It is found that the modified electron-plasma mode, in addition to the usual ordinary and extraordinary modes, is excited as Cerenkov radiation at frequencies greater than the plasma frequency. McKenzie [5,6] investigated the Cerenkov radiation in a magnetoionic medium with an emphasis on the evaluation of radiation losses suffered by a charged particle moving through the plasma and on its application to the generation of low-frequency electromagnetic radiation in the upper atmosphere due to the passage of streaming charged particles. In these works, Cerenkov excitation of the whistler and ion-cyclotron waves, and their critical dependence on some particular characteristic wave speed relative to the particle's velocity component parallel to the magnetic field are studied. The intensity of the emitted radiation has been found to decrease with the decrease of the particle speed along the magnetic field.

The phenomena of Cerenkov and related emissions within the upper atmosphere of the Earth have been well-understood from the theoretical as well as experimental standpoint in a comprehensive manner through the works of many authors [5,8–21].

When a plasma wave is slowed and approaches the electron thermal velocity, its interaction with the thermal electron increases and gives rise to Landau damping attaining eventually Cerenkov resonance conditions. Cerenkov condition becomes significant when the phase velocity approaches the electron thermal velocity which is comparable for pure and modified Alfvén waves in the ELF range of the ionosphere around 300 km to 500 km heights.

Etcheto and Gendrin [12], De *et al* [21] and Singh [22] explored the possibility of VLF Cerenkov emission in the ionosphere by electron beams within the ionosphere. The study of resonance cone phenomena in plasmas is widely known [23,24]. Balmain [25] has given an exhaustive list of references on the subject.

In this paper, a model calculation has been made to investigate the radiation characteristics of a point charge moving with uniform velocity through the ionospheric plasma. The influences of time-varying geomagnetic irregu-

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larities caused due to VLF hisses, whistler-mode propagation enhanced by auroral electrons and due to other physical processes within the medium as well as the motion of heavy ions have been taken into account. In the mathematical analysis, coupled equations are obtained. Using suitable transformation for decoupling, the equations are solved. The emitted radiation is found to be consisted of two modes. The dispersion relation of these modes may be analysed to examine the effects of time-varying irregularities and motion of heavy ions on the low frequency spectrum of the two modes. The direction of Cerenkov ray for different values of propagation frequency, normalized by plasma frequency, has been shown graphically.

2. Mathematical formulation

In the stated model, a line charge will be assumed to move with a uniform velocity u along the z -direction. Maxwell's equations under Fourier transform yield

$$\nabla \times E(r, \omega) = j\omega H(r, \omega), \quad (1)$$

$$\nabla \times H(r, \omega) = -j\omega \left(\bar{\epsilon} \right) E + \hat{z} J(r, \omega), \quad (2)$$

$\left(\bar{\epsilon} \right)$ is the dielectric tensor of the medium in presence of electrons and ions. To deduce the dielectric tensor, the non-relativistic Lorentz force equation has been taken as

$$\frac{\partial v}{\partial t} = \frac{e}{m} \left[E + \frac{v \times B}{c} \right] - \eta v, \quad (3)$$

η is the collision factor and B is the geomagnetic field. The other symbols have their usual significance.

As the time-varying irregularities within the ionosphere give rise to time-varying magnetic field over and above the static geomagnetic field, the effective magnetic field B can be written as

$$B = B_0 + B_1 \exp(j\omega_0 t), \quad (4)$$

B_0 is the earth's magnetic field which is taken to be in the z -direction and $B_1 \exp(j\omega_0 t)$ is due to irregularities with frequency ω_0 .

Introducing dielectric polarization $P (=Ner)$ and the relation (4), the eq., (3) under Fourier transform through the aid of Faltung theorem yields

$$eE = \frac{m}{N} (j\omega\eta - \omega^2) P - \frac{j\omega P}{Ne} \times [B_0 + B_1 \delta(\omega - \omega_0)], \quad (5)$$

where δ is the Dirac delta function.

Writing in (5) $P = \sigma E$ and using $D = \epsilon E = E + 4\pi P$, the expression of dielectric tensor can be deduced. Including the contribution of ions also, the components of dielectric tensor have been obtained as

$$\begin{aligned} \epsilon_{11} &= 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} + j\eta \frac{\omega_{pe}^2 (\omega^2 + \omega_{ce}^2)}{\omega (\omega^2 - \omega_{ce}^2)^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} \\ &\quad + j\eta \frac{\omega_{pi}^2 (\omega^2 + \omega_{ce}^2)}{\omega (\omega^2 + \omega_{ce}^2)^2} - \frac{2\omega_{pe}^2 \omega_{ce} \omega_{ci}}{(\omega^2 - \omega_{ce}^2)^2} \delta(\omega - \omega_0), \\ \epsilon_{12} &= - \frac{\omega_{pe}^2 \omega_{ce}}{\omega (\omega^2 - \omega_{ce}^2)} - \frac{2\eta \omega_{pe}^2 \omega_{ce}}{(\omega^2 - \omega_{ce}^2)^2} - \frac{j\omega_{pi}^2 \omega_{ci}}{\omega (\omega^2 - \omega_{ci}^2)} \\ &\quad - \frac{2\eta \omega_{pi}^2 \omega_{ci}}{(\omega^2 - \omega_{ci}^2)^2} + \frac{j\omega_{pe}^2 \omega_{ce} (\omega^2 + \omega_{ce}^2)}{\omega (\omega^2 - \omega_{ce}^2)^2} \delta(\omega - \omega_0), \\ \epsilon_{13} &= \frac{\omega_p^2 (\omega_{ce} \omega_{cx} - j\omega \omega_{cy})}{\omega^2 (\omega^2 - \omega_{ce}^2)} \delta(\omega - \omega_0), \\ \epsilon_{21} &= -\epsilon_{12}, \\ \epsilon_{22} &= \epsilon_{11}, \\ \epsilon_{23} &= \frac{\omega_p^2 (\omega_{ce} \omega_{cy} + j\omega \omega_{cx})}{\omega^2 (\omega^2 - \omega_{ce}^2)} \delta(\omega - \omega_0), \\ \epsilon_{31} &= \frac{\omega_p^2 (\omega_{ce} \omega_{cx} + j\omega \omega_{cy})}{\omega^2 (\omega^2 - \omega_{ce}^2)} \delta(\omega - \omega_0), \\ \epsilon_{32} &= \frac{\omega_p^2 (\omega_{ce} \omega_{cy} - j\omega \omega_{cx})}{\omega^2 (\omega^2 - \omega_{ce}^2)} \delta(\omega - \omega_0), \\ \epsilon_{33} &= 1 - \frac{\omega_{pe}^2}{\omega^2} - j\eta \frac{\omega_{pi}^2}{\omega} \end{aligned}$$

where

$$\omega_{pe}^2 = \frac{4\pi N_e e^2}{m_e}, \quad \omega_{pi}^2 = \frac{4\pi N_i e^2}{m_i}$$

$$\omega_{ce} = \frac{eB_0}{m_e c}, \quad \omega_{ci} = \frac{eB_0}{m_i c},$$

$$\omega_{cx} = \frac{eB_{1x}}{m_e c}, \quad \omega_{cy} = \frac{eB_{1y}}{m_e c}, \quad \omega_{cz} = \frac{eB_{1z}}{m_e c}$$

m_e is the mass of an electron; m_i , the mass of a heavy ion; N_e , the electron number density; N_i , the heavy ion number density; ω_{pe} , the electron plasma frequency; ω_{pi} , the ion plasma frequency; ω_{ce} , the electron gyrofrequency and ω_{ci} is the ion gyrofrequency. ω_{cx} , ω_{cy} and ω_{cz} are the components of the electron gyrofrequency due to time-varying irregularities. The influence of time-varying irregularities on ions has been neglected.

The external magnetic field is considered to be traversed along the z -direction, where ρ , ϕ and z would be assumed to form a cylindrical co-ordinate system. The expression for the point charge moving along the direction of the external magnetic field is given by

$$q = q_0 \frac{\delta(\rho)}{2\pi\rho} \delta(z - ut). \quad (6)$$

The corresponding current density can be written as

$$J(r, t) = \hat{z} q_0 u \frac{\delta(\rho)}{2\pi\rho} \delta(z - ut), \quad (7)$$

r is the position vector of the point charge in the (ρ, ϕ, z) system. The Fourier transform of (2) yields

$$J_z(r, \omega) = q_0 \frac{\delta(\rho)}{2\pi\rho} e^{j\omega z/u}. \quad (8)$$

The field components are independent of ϕ , but dependent on z through the phase factor $e^{j\omega z/u}$ and can be represented as

$$\left. \begin{aligned} \bar{E}(r, \omega) &= E(\rho, \omega) e^{j\omega z/u}, \\ \bar{H}(r, \omega) &= H(\rho, \omega) e^{j\omega z/u}. \end{aligned} \right\} \quad (9)$$

The current density

$$J(r, \omega) = J(\rho, \omega) e^{j\omega z/u}. \quad (10)$$

From eqs. (1), (2), (9) and (10), the quantities E_ρ , E_z , H_ρ , H_z can be expressed in terms of H_ϕ and E_ϕ . The coupled equations for E_ϕ and H_ϕ are obtained as

$$\begin{aligned} \frac{\partial}{\partial \rho} \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho E_\phi) \right] + \left[\omega^2 \epsilon_{11}^2 - \epsilon_{22}^2 + \epsilon_{11} \epsilon_{12} \right] \omega^2 \\ = \frac{j\omega^2 \epsilon_{22}}{c^2 \epsilon_{11}} H_\phi, \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{\partial}{\partial \rho} \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho H_\phi) \right] + \left[\frac{\omega^2}{c^2} \epsilon_{33} - \frac{\omega^2}{u^2} \frac{\epsilon_{33}}{\epsilon_{11}} + \left(1 + \frac{\epsilon_{12}}{\epsilon_{11}} \right) \right] H_\phi \\ = \frac{j\omega^2 \epsilon_{12} \epsilon_{33}}{c^2 \epsilon_{11} u} E_\phi - \frac{\partial}{\partial \rho} J_z. \end{aligned} \quad (12)$$

The coupled wave eqs. (11) and (12) can be solved by Hankel transform technique [4,26]. The transform of order 1 can be defined as

$$\bar{E}_\phi(\zeta, \omega) = \int_0^\infty E_\phi(\rho, \omega) J_1(\zeta \rho) \rho d\rho, \quad (13)$$

$$E_\phi(\rho, \omega) = \int_0^\infty \bar{E}_\phi(\zeta, \omega) J_1(\zeta \rho) \zeta d\zeta, \quad (14)$$

$$\bar{H}_\phi(\zeta, \omega) = \int_0^\infty H_\phi(\rho, \omega) J_1(\zeta \rho) \rho d\rho, \quad (15)$$

$$H_\phi(\rho, \omega) = \int_0^\infty \bar{H}_\phi(\zeta, \omega) J_1(\zeta \rho) \zeta d\zeta, \quad (16)$$

J_1 is the first-order Bessel function.

Applying (13) and (15) in eqs. (11) and (12) and solving for $\bar{E}_\phi(\zeta, \omega)$ and $\bar{H}_\phi(\zeta, \omega)$, one can get

$$\bar{E}_\phi(\zeta, \omega) = \frac{j q_0 \omega^2 \zeta \epsilon_{22}}{4\pi u \epsilon_{11} \Delta} \quad (17)$$

and

$$\bar{H}_\phi(\zeta, \omega) = \frac{q_0 \zeta \left[(k_1^2 - \zeta^2) + \eta^2 \right]}{4\pi \Delta} \quad (18)$$

where

$$\begin{aligned} \Delta &= (k_1^2 - \zeta^2) (k_2^2 - \zeta^2) - \frac{2\omega^4 \epsilon_{21}^2 \epsilon_{33}}{c^2 u^2 \epsilon_{11}^2} \\ &= (\zeta^2 - k_0^2) (\zeta^2 - k_e^2), \end{aligned} \quad (19)$$

$$k_1^2 = \frac{\omega^2}{c^2} \epsilon_{33} - \frac{\omega^2}{u^2} \frac{\epsilon_{33}}{\epsilon_{11}} \left(1 + \frac{\epsilon_{12}}{\epsilon_{11}} \right),$$

$$k_2^2 = \frac{\omega^2}{c^2} \frac{\epsilon_{11}^2 - \epsilon_{22}^2 + \epsilon_{11}\epsilon_{12}}{2} - \frac{\omega^2}{u^2} \quad (20)$$

$$\eta^2 = (\omega^2 \epsilon_{12}^2) / (cu \epsilon_{11}^2).$$

Introducing (17), (18), (19) and (20) in (14) and (16), one can get the solutions after integration as

$$E_\phi(\rho, \omega) = E_{\phi 0}(\rho, \omega) + E_{\phi e}(\rho, \omega),$$

$$E_\phi(\rho, \omega) = -\frac{j\omega^2 \epsilon_{12}}{u \epsilon_{11} (k_0^2 - k_e^2)} H_{\phi 0}(\rho, \omega) - \frac{j\omega^2 \epsilon_{22}}{u \epsilon_{11} (k_e^2 - k_0^2)} H_{\phi e}(\rho, \omega), \quad (21)$$

$$H_\phi(\rho, \omega) = H_{\phi 0}(\rho, \omega) + H_{\phi e}(\rho, \omega) \quad (22)$$

$$H_\phi(\rho, \omega) = -\frac{j q_0}{6} \hat{k}_0 \frac{k_0^2 - k_e^2}{k_0^2 - k_e^2} H_1^{(1)}(\hat{k}_0, \rho) + \frac{j q_0}{6} \hat{k}_e \frac{k_e^2 - k_0^2}{k_0^2 - k_e^2} H_1^{(1)}(\hat{k}_e, \rho), \quad (23)$$

$$\hat{k}_0 = \pm k_0, \hat{k}_e = \pm k_e.$$

Here, $H_1^{(1)}$ is the Hankel function of first kind. The two possible modes are denoted by subscripts 0 and e.

3. Dispersion relation

To obtain the frequency spectrum of radiated energy, the frequency ranges where k_0 and k_e are positive should be determined. The functional dependence of k_0 and k_e on ω can be secured from (19) and (20) as

$$k_{0,e}^2 = \frac{\omega^2}{6u^2} \left[-s_1 \pm \sqrt{s_1^2 - 4s_2^2} \right], \quad (24)$$

where

$$s_1 = \left[1 + \frac{\epsilon_{33}}{\epsilon_{11}} - \frac{u^2}{c^2 \epsilon_{11}} (\epsilon_{11}^2 - \epsilon_{12}^2 + \epsilon_{11}\epsilon_{21}) \right], \quad (25)$$

$$s_2 = \left[\frac{\epsilon_{33}}{\epsilon_{11}} - 2\epsilon_{33} \left(\frac{u}{c} \right)^2 + \left(\frac{u}{c} \right)^4 \frac{\epsilon_{33}}{\epsilon_{11}} (\epsilon_{11}^2 - \epsilon_{12}^2 + \epsilon_{33}^2) \right]. \quad (26)$$

Using (25), (26) and the values of the elements of the dielectric tensor, one can obtain the dispersion relation from (24).

4. Direction of Cerenkov radiation

The powers radiated in the ordinary and extra-ordinary modes per unit length and per unit frequency are given by

$$I_0(\omega) = 2\pi\rho \langle s_{\rho 0} \rangle$$

and

$$I_e(\omega) = 2\pi\rho \langle s_{\rho e} \rangle, \quad (27)$$

$\langle s_{\rho 0} \rangle$ and $\langle s_{\rho e} \rangle$ are the radial components of time averaged Poynting vectors for the two modes. $I_0(\omega)$ and $I_e(\omega)$ have been obtained as

$$I_0(\omega) \equiv \pm \frac{q_0^2 k_0^2}{32\omega \epsilon_{33}} \frac{k_0^2 - k_e^2 + \eta^2}{k_0^2 - k_e^2 + \eta^2} \quad (28)$$

$$I_e(\omega) \equiv \pm \frac{q_0^2 k_0^2}{32\omega \epsilon_{33}} \frac{k_e^2 - k_0^2 - \eta^2}{k_e^2 - k_0^2 - \eta^2} \quad (29)$$

The upper and lower signs correspond to the upper and lower signs of (23). These must be chosen correctly so that $I_0(\omega) > 0$ and $I_e(\omega) > 0$, within the frequency range where the radiation is possible.

For Cerenkov ray, the z-component of the time-averaged Poynting vector is necessary. This yields, after some algebraic simplifications, for the ordinary wave as

$$\langle s_{z0} \rangle = \frac{1}{2\pi\rho} \frac{q_0^2 k_0^2 (k_0^2 - k_e^2 + \eta^2)}{32c \epsilon_{33} (k_0^2 - k_e^2 + \eta^2)^2} \left[\frac{\omega^2 \epsilon_{11}}{u^2} (k_0^2 - k_e^2 + \eta^2) - \frac{\omega^4 \epsilon_{12}^2}{c^2 \epsilon_{11}} (\epsilon_{11} \epsilon_{12} + \epsilon_{11}^2 - \epsilon_{12}^2) \frac{k_e^2 - k_0^2}{\epsilon_{33}} \right] \quad (30)$$

For the extra-ordinary wave, the expression of $\langle s_{ze} \rangle$ can be obtained by interchanging k_0 and k_e in (30). The angle θ between the Cerenkov ray and the direction of motion of the source is given by

$$\tan \theta_0 = \frac{\langle s_{\rho 0} \rangle}{\langle s_{z0} \rangle}$$

and

$$\tan \theta_e = \frac{\langle s_{pe} \rangle}{\langle s_{ze} \rangle}, \quad (31)$$

where θ_0 and θ_e refer to ordinary and extraordinary waves.

5. Discussion

In the analysis, the influences of time-varying irregularities which introduce perturbation over the ambient value of the geomagnetic field and the influence of the motion of the heavy ions are taken into account through the dielectric tensor. The perturbed magnetic field is associated with VLF hisses, the incoherent whistler radiation by the beam of precipitating auroral electrons, micropulsations of various types. The effect of motion of heavy ions is important in the *F*-region of the ionosphere. Thus, the contributions of these effects on the frequency spectrum of Cerenkov radiation from the upper atmosphere may be examined from the present analyses.

The direction of the Cerenkov radiation (θ_e) for the extra-ordinary wave with the variation of propagation frequency, normalized by the plasma frequency, is evaluated numerically using (27), (30) and (31), and presented graphically by the continuous line curves for three different values of ω/ω_c (Figure 1). The value of ω_c/ω_p is chosen

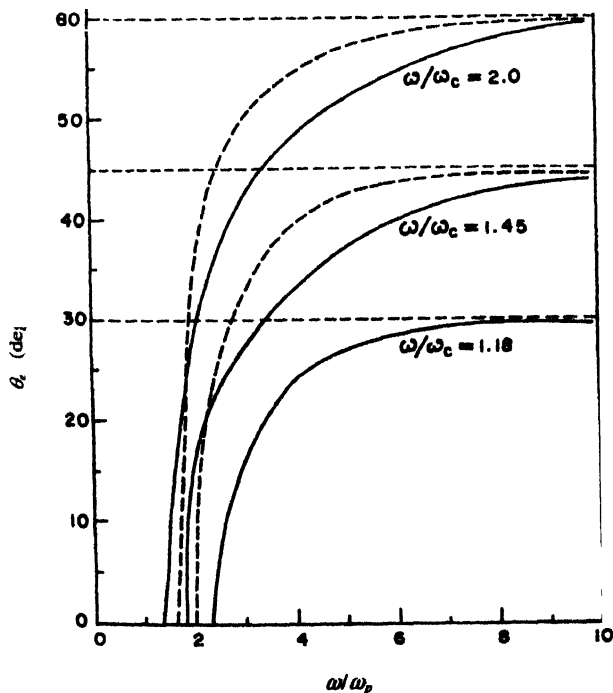


Figure 1. The variation of the direction of Cerenkov radiation (θ_e) for the extra-ordinary wave with the variation of propagation frequency, normalized by the plasma frequency (ω/ω_p), for three different values of ω/ω_c . The continuous line curves show the results of the present analysis while the dotted curves are due to an earlier work [4].

to be unity in all the calculations for the direction of Cerenkov ray. CIRA 72 data [27], IRI data [28] and the data obtained from C_2 -recorder at Haringhata field station [29] are used in the numerical analyses. The lower end of the frequency spectra are radiated close to the direction of motion of the source. With the increasing frequency, the angle of radiation is also increased. It is seen that there will be no Cerenkov rays which are radiated at an obtuse angle from the direction of motion of the source. The present result agrees with the results of an earlier work [4] shown by the dotted curves. The partial disagreement may be due to the inclusion of the effects of time-varying geomagnetic irregularities and the motion of heavy ions which are not considered in the earlier work [4].

The results can be made useful to investigate the generation mechanism of observed VLF hisses at high altitudes [11,30]. Although an idealized plasma model has been considered, it is expected that the present study may provide some physical insight into the possible radiation from a moving charged particle in the anisotropic ionosphere traversed by the geomagnetic field and time-varying irregularities.

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